Rupture en fatigue de structures composites stratifiés

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Fatigue rupture of laminated composite structures

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Résumé

Un modèle basé sur la mécanique continue de l'endommagement et un critère de rupture non local ont été développés pour étudier le comportement à rupture de structures stratifiées. Dans ce travail, le modèle développé précédemment dans le cas de chargements statiques a été étendu à la fatigue. Le modèle d'endommagement décrit les effets de la dégradation du composite dans des directions transversales et de cisaillement ainsi que les déformations anélastiques dans la direction de cisaillement. Le critère non local permet de décrire la rupture dans la direction des fibres en présence de concentrations de contrainte. Le modèle et le critère non local ont été étendus à la fatigue et implémentés dans Abaqus. Enfin, des exemples sont présentés dans le cas de stratifiés tissés déséquilibrés verre/epoxy et de plaques trouées sous chargements cycliques de tension-tension.

Abstract

Continuum Damage Mechanics (CDM) model and non local failure criterion have been developed to study the mechanical response of laminated structures until the first ply failure. In this work, the model previously developed in the case of static loading was extended to fatigue loading conditions. The damage model describes the effects of matrix degradation in both transverse and shear directions and the inelastic strains in the shear direction. The non local failure criterion takes into account the effects due to matrix damage and stress concentrations for predicting the failure in the fibre direction. The model and the failure criterion were implemented in the Finite Element Code Abaqus. Applications to glass/epoxy unbalanced woven ply laminates were performed in case of open hole plates under tension-tension cyclic loading.

Mots Clés : stratifié, endommagement, fatigue, critère de rupture, concentration de contrainte
Keywords : Laminate, Damage, Fatigue, Failure criterion, Stress concentration

1. Introduction

Static and fatigue loading conditions trigger same damage mechanisms in composite materials, such as matrix cracking, fibre/matrix debonding, transverse failure and fibre failure mechanisms [1,2]. Models based on Continuum Damage Mechanics (CDM) allow describing at the ply scale the effects of these various damage phenomena. Such as a model was developed to describe the behaviour of unbalanced woven ply laminates under static and fatigue loading [3]. The effects of matrix degradations on the stiffness reduction were modelled by describing the damage evolution with a non linear cumulative law which took into account the maximal load and the loading amplitude. The model described the effects of the damage evolution on the warp, weft and shear directions. Inelastic strains in the shear direction were also modelled.
Fibre failure mechanism has usually catastrophic effects which lead quickly to the complete failure of the laminate. The brittle failure due to fibre failure was thus studied and stress concentration effect was identified. A non local failure criterion was developed to predict the fibre tensile failure in laminates with stress concentrations [4]. The models of behaviour combined with the non local failure criterion were applied successfully to various woven ply laminated structures in the case of static loading [5].
This paper is focused on the failure of laminates with stress concentrations in the case of fatigue loading. Experimental tests led to observe a strong influence of the matrix damage on the fibre

tensile failure. A new failure criterion was defined to take into account the effects of matrix damage and stress concentration on the failure of the laminated structures in the fibre direction. The paragraph 2 reminds the assumptions and laws which characterise the damage model. The development of a new fibre tensile failure criterion in the case of fatigue loading is described in the paragraph 3. Comparisons between experimental data and numerical predictions on glass/epoxy unbalanced woven ply laminates are summarized in the paragraph 4.

2. Modelling of the damageable behaviour of woven ply

2.1. Assumptions

A model based on the Continuum Damage Mechanics (CDM) has been developed to describe the damageable behaviour of composite material at the ply scale [2]. The damage is assumed to be uniform in the thickness of the ply.

The laminate is modelled as a stacking sequence of UD plies. That means in the case of woven ply that each ply is replaced by a two-UD-plies-laminate. The example of a 0° unbalanced woven ply with δ % fibres in the warp direction and (100-δ) % fibres in the weft direction is studied here. The woven ply is modelled by two UD plies as:

- One UD ply oriented at 0° whose thickness is equal to δ % of the whole woven ply thickness
- One UD ply oriented at 90° whose thickness is (100-δ) % of the whole thickness.

The modelling of both static and fatigue loadings with the same model is allowed by the use of a non-linear cumulative law which describes the damage evolution according to the maximal load and the amplitude of the cyclic loading. A kinematic hardening model was used to describe the inelastic strains [2].

2.2. Damage evolution in the UD ply

In the fibre direction, the UD ply shows linear elastic behaviour in response to tension loading until the final brittle failure. In the transverse and shear directions, the behaviour is non linear due to the damages which lead to a decrease in stiffness. The damage kinematics was described by three internal damage variables:

- d_1, whose evolution represents the linear elastic behaviour and the brittle failure of the fibres observed in response to tension loading applied in the longitudinal direction
- d_2, which models the lost of transverse stiffness resulting from transverse and shear loadings
- d_{12}, which describes the decrease in shear stiffness due to transverse and shear loadings.

The gradual development of both damage d_2 and d_{12} depends on the tension load as well as on the shear load, which generates the matrix cracks. Assuming the existence of plane stresses and small perturbations, the strain energy in the ply can be written as follows [1,6]:

$$E_D^{ps} = \frac{1}{2}\left[\frac{\langle\sigma_1\rangle_+^2}{E_1^0(1-d_1)} + \frac{\langle\sigma_1\rangle_-^2}{E_1^0} + \frac{\langle\sigma_2\rangle_+^2}{E_2^0(1-d_2)} + \frac{\langle\sigma_2\rangle_-^2}{E_2^0} - 2\frac{\nu_{12}^0}{E_1^0}\sigma_1\sigma_2 + \frac{\sigma_{12}^2}{G_{12}^0(1-d_{12})}\right] \quad \text{(Eq. 1)}$$

where < . >$_+$ is the positive part and < . >$_-$ is the negative part. The tension energy and the compression energy are split in order to describe the unilateral nature of the damage process due to the opening and closing of the cracks.

Static loading

In the case of static loading, the thermodynamic forces associated with the internal tension and shear variables d_1^s, d_2^s and d_{12}^s are defined as follows:

$$\begin{cases} Y_{d_i^s} = \dfrac{\partial E_D^{ps}}{\partial d_i^s} = \dfrac{\langle \sigma_i \rangle_+^2}{2E_i^0 (1-d_i^s)^2} \qquad with\ i = 1, 2 \\ Y_{d_{12}^s} = \dfrac{\partial E_D^{ps}}{\partial d_{12}^s} = \dfrac{(\sigma_{12})^2}{2G_{12}^0 (1-d_{12}^s)^2} \end{cases} \qquad \text{(Eq. 2)}$$

The development of internal variables depends on these thermodynamic forces and more precisely, on their maximum values during the loading history. Under tension loading conditions, d_1^s has to develop suddenly to model the brittle behaviour in the fibre direction. So, d_1^s is defined as:

$$\begin{cases} d_1^s = 0 \quad if\ Y_{d_1^s} < Y_1^{\max} \\ d_1^s = 1 \quad if\ Y_{d_1^s} \geq Y_1^{\max} \end{cases} \qquad \text{(Eq. 3)}$$

where $Y_1^{\max}$ is the parameter defining the ultimate force in the fibre direction.

The tension/shear coupling during the development of d_2^s and d_{12}^s is accounted for by the following equivalent thermodynamic force:

$$Y_{eq} = a\left(Y_{d_2^s}\right)^n + b\left(Y_{d_{12}^s}\right)^m \qquad \text{(Eq. 4)}$$

where a, b, m and n are material parameters specifying the tension/shear coupling. The evolution law for the damage is written as:

$$d_2^s = \left\langle 1 - e^{-\left(Y_{eq} - Y_0^s\right)} \right\rangle_+ \qquad \text{(Eq. 5)}$$

$$d_{12}^s = c\ d_2^s \qquad \text{(Eq. 6)}$$

where the constant parameter Y_0^s corresponds to the threshold value of the development of d_2^s (which ranges from 0 to 1) and d_{12}^s is proportional to d_2^s.

Fatigue loading

In the case of fatigue loading [3], damage evolution depends on the maximal load $Y_{d_i^f}$ and the amplitude of the loading $\Delta Y_{d_i^f}$ during a cycle. The damage evolution law is then written as:

$$\frac{\partial d_2^f}{\partial N} = (1-d_2)^\gamma \left\langle a_f \left(Y_{d_2^f}\right)^{\beta_1} \left(\Delta Y_{d_2^f}\right)^{\beta_2} + b_f \left(Y_{d_{12}^f}\right)^{\beta_3} \left(\Delta Y_{d_{12}^f}\right)^{\beta_4} - Y_0^f \right\rangle_+ \qquad \text{(Eq. 7)}$$

where: Y_0^f is the threshold value of the development of d_2^f

$$\Delta Y_{d_2^f} = \frac{\left(\left\langle \sigma_2^{\max} \right\rangle_+ - \left\langle \sigma_2^{\min} \right\rangle_+\right)^2}{2\, E_2^0 \left(1 - d_2^f\right)^2} \quad and \quad \Delta Y_{d_{12}^f} = \frac{\left(\sigma_{12}^{\max} - \sigma_{12}^{\min}\right)^2}{2\, G_{12}^0 \left(1 - d_{12}^f\right)^2} \qquad \text{(Eq. 8)}$$

Cumulative damage evolution law

The cumulative damage evolution law is defined as follow where the damage variables in the transverse and the shear directions, denoted respectively d_2 and d_{12}, are obtained by adding the terms due to static and fatigue loadings.

$$d_2 = d_2^s + d_2^f \qquad \text{(Eq. 9)}$$

$$d_{12} = c\, d_2 \qquad \text{(Eq. 10)}$$

3. Fibre tensile failure criterion

3.1. Damage influence on stress distribution

The damage influence on the stress distribution in the ply and thus on the laminate failure was studied. In the case of woven ply, damage occurs in both longitudinal and transverse directions even if the loading direction is parallel to fibres [4,5]. For example in a 0° ply loaded in tension, damage occurs in the transverse direction, which means matrix cracks propagate along the weft fibres, but also in the longitudinal direction. Due to weft fibres, strains in the transverse direction are blocked that leads to generate tensile stresses in this direction. Then, initiation of cracks occurs along the warp fibres. During the loading, crack density increases in both warp and weft directions. In the transverse direction, the damage leads to a decrease of the stiffness. In the longitudinal direction, the damage disrupts the load transfer between fibres. Fibre failure can then occur even if the maximal stress usually measured in the case of homogeneous tension test is not reached. This phenomenon was not observed in the case of static loading due to the weak crack density compared to the case of fatigue loading where the damage can reach a very high level.
Experimental tests were performed to study this phenomenon which can led to premature failure of laminate. Specifics tubes were manufactured with Glass/Epoxy unbalanced woven ply. The tube shape was studied so that the strains field was homogeneous in the central area. The geometry of the specimen is presented in Figure 1. The lay-up in the central area was $(0)_3$. Torsion cyclic loading was applied to the tubes to generate matrix damage in both longitudinal and transverse directions. The rotation was limited to avoid applying load on fibres. Stress/Strain curves were plotted to evaluate the decrease of stiffness which characterise the evolution of the damage as we can see on Figure 2. Various levels of damage were obtained according to the number of cycles applied. Then, the tubes were loaded in static tension until the final failure to estimate the residual strength of the warp fibres. Figure 3 shows the influence of the damage on the strength at failure in the fibre direction. The Load/Displacement curves were plotted for safe and damaged tubes. Fibre damage would affect the stiffness of the specimen. The slope of the different curves is relatively similar that underlines the fatigue tests generated few fibre damages.
Tests showed the strong influence of the matrix damage on the fibre failure. Because of matrix cracking and matrix/fibre debonding, the load transfer between fibres could not be fully. Failure of the ply occurred although the stress in the fibre direction was lower than the tensile failure strength.

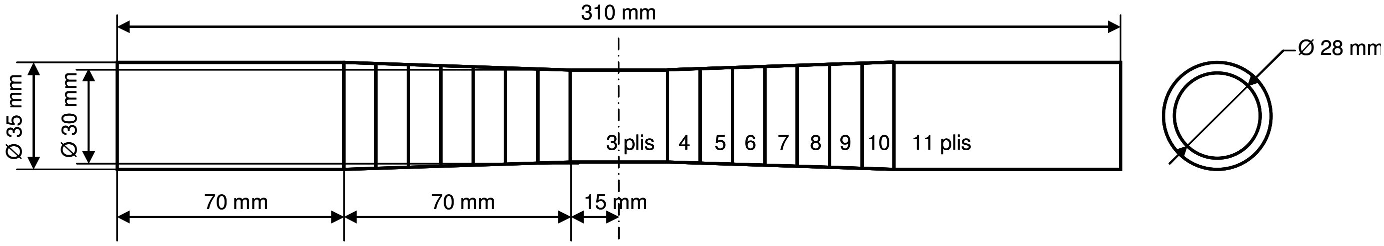

Fig.1: Geometry of the tube

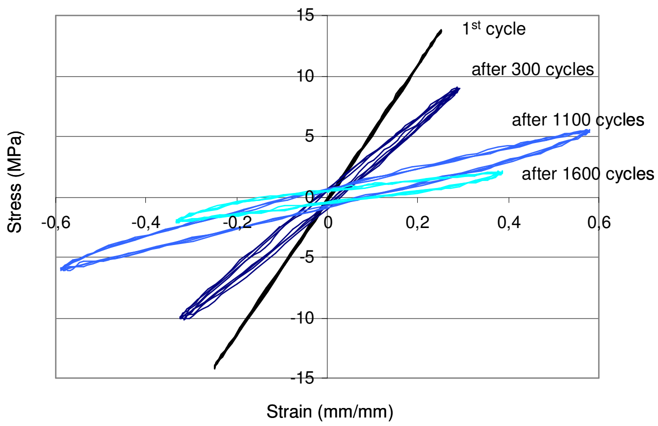

Fig.2: Stiffness decrease due to the damage evolution during the cycles

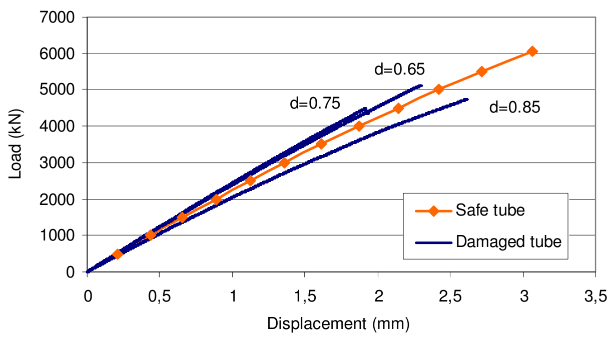

Fig.3: Tension tests on safe and damaged tubes

3.3. Non local failure criterion

The model was defined at the UD ply scale. In the case of woven ply, the model assumes that a woven ply has the same behaviour than a (0,90) UD laminate. This assumption offers the great advantage to study clearly the warp and the weft directions of woven ply. For each UD ply modelled, the matrix damage is evaluated and its influence on the fibres strength can be taken into account with the following criterion:

$$Y_{d_1} \leq Y_{d_1}^{\max}(d_2) \tag{13}$$

where the thermodynamic force Y_{d_1} is proportional to the longitudinal stress (Refer to eq.(2)) and d_2 is the transverse damage in the UD ply. $Y_{d_1}^{\max}$ evolves sharply between two values according to a threshold value of d_2 as it was shown in Figure 4.

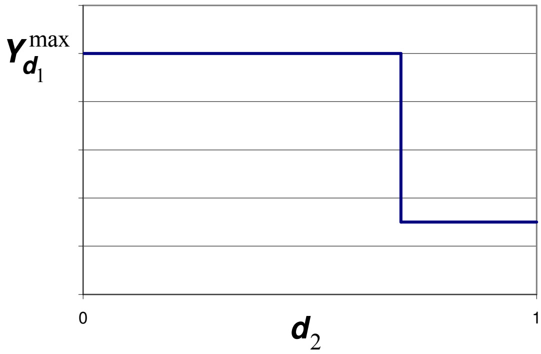

Fig.4: $Y_{d_1}^{\max}$ *evolution law according to the level of the transverse damage d_2*

In the case of static loading, the level of damage does not usually reach the threshold and the criterion (13) is equivalent to a maximal stress criterion. But in the case of fatigue loading with a high number of cycles, the level of the damage cannot be neglected.

Previous studies led to observe a strong underestimation of the failure strength when a local criterion was used to predict the failure of laminated structure [4]. An original approach based on a *Fracture Characteristic Volume* (FCV) has been developed to predict the failure of laminated structures with stress concentrations [4,5]. The FCV was a cylinder defined at the ply scale as the volume $V = hS$, where h corresponds to the thickness of the ply and S is the in-plane area (Fig. 5).

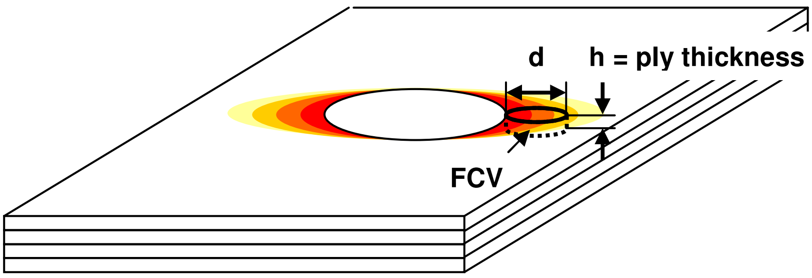

Fig.5: Non local criterion based on Fracture Characteristic Volume (FCV)

Non local fracture criterion was defined in the case of static loading as:

$$\overline{Y_{d1}} = \frac{\left(\frac{1}{V} \int_V \langle \sigma_1 \rangle_+ \, dV \right)^2}{2\, E_1^0} \qquad \textit{and} \qquad \overline{Y_{d1}} < Y_{d_1}^{\max} \qquad \text{(Eq. 14)}$$

where $\overline{Y_{d1}}$ is the average thermodynamic force associated to the damage variable in the longitudinal direction d_1 and $Y_{d_1}^{\max}$ is a material property which needs to be identified.

The extension to fatigue loading led to modify the criterion and take into account the influence of matrix damage on the tensile fibre failure. The criteria (Eq. 13) and (Eq. 14) have become:

$$\begin{cases} \overline{Y_{d_1}} = \dfrac{\left(\dfrac{1}{V_f} \int_{V_f} \langle \sigma_1 \rangle_+ \, dV \right)^2}{2\, E_1^0} \\ \overline{d_2} = \dfrac{1}{V_f} \int_{V_f} d_2 \, dV \end{cases} \quad and \quad \overline{Y_{d_1}} < Y_{d_1}^{\max}(\overline{d_2}) \qquad \text{(Eq. 15)}$$

The same volume V_f was used to evaluate both average values of the thermodynamic force and the damage and can be different from the *FCV* obtained in static.

4. Application to glass/epoxy unbalanced woven ply

4.1. Identification of the material properties

The model was applied to Glass/Epoxy unbalanced woven ply. The woven ply was modelled by two UD plies with different thicknesses to take into account the different proportions of fibres in the warp and the weft directions. The elastic properties of both UD plies were evaluated from the elastic properties of the woven ply. The parameters of the damage law were equal for both UD plies in the case of static loading [2]. However, in the case of fatigue loading, the influence of the ply thickness on the damage evolution cannot be neglected. So, the damage properties were defined separately for the thin and the thick ply. The identification of the material properties specifically required by the model is briefly reminded here. More details can be found in [3].

Static damage law

The damage laws in the case of static loadings were defined by equations (3) to (6). In the case of the unbalanced woven ply, three tension tests were necessary to identify the parameters of the damage model: $Y_1^{\max}$ was defined with a tension test on a $(0)_8$ laminate, *a*, *m* and Y_0^s, in a $(90)_8$ test, and *b*, *c* and *n* in a $(22)_8$ test.

Fatigue damage law

The damage law was defined by the equation (7). The parameters were identified from S-N curves obtained for (0), (90) and (45) laminates. The (0) and the (90) curves provided the value of the parameters a_f, β_1 and β_2 for each UD ply which models respectively the behaviour of the woven ply in the warp direction and the weft direction. The parameters b_f, β_3 and β_4 were identified from the (45) laminate results. Finally γ was evaluated from the (0) curve and Y_0^f was defined equal to 0 which means there was not any threshold and the fatigue damage started at the first cycle.

4.2. Identification of the failure criteria parameters

Two failure modes were identified during the experimental tests:

- the failure due to the localisation of the damage which leads to the instability of the structure and then, the complete failure of the specimen. This failure mode appeared usually for out-of-axis laminates as (45) laminate.

- the failure of the fibres which leads quickly to the final failure of the specimen. Strong influence of damage and stress concentration on this failure mode was observed. The criterion (15) was then defined.

The first failure mode is defined by the instability of the structure which leads to stop the computation. No parameter has to be identified.
The second failure mode is defined by three parameters:

- $Y_{d_1}^{\max}$, which characterises the failure strength in the fibre direction and is function of
- d_2, the level of damage which leads to the failure strength decrease
- V, the size of the *Fracture Characteristic Volume* in static
- V_f, the size of the *Fracture Characteristic Volume* in fatigue

The identification of the failure criterion was based on the evolution of the failure strength in the fibre direction according to the level of damage in the ply. The evolution of the criterion was chosen as presented in Figure 4. The threshold of the damage d_2 and the value of the strength at failure $Y_{d_1}^{\max}$ before and after the threshold were defined from the tests on tube described previously (see Fig. 1-3).
The identification of the *FCV* has been defined from fatigue test with a high number of cycles in order to take into account the effect of the damage which leads to a stress redistribution around the hole. As explained in [4], only one test on a specimen with stress concentration is required to identify the size of the *FCV*. A test at 50% of the maximal load on a (QI) open hole plate was chosen. The comparison between the experimental data and the numerical simulation provided a diameter of the *FCV* equals to 1.6 mm. In the case of static loading, the diameter was equal to 0.6 mm for the same materials [5].

4.3. Open hole plate

Open hole plate were manufactured with Glass/Epoxy unbalanced woven ply. The geometry of the plate is presented in Figure 6. Two different laminates were studied $(0,-45,+45,90)_S$, noted as (QI), and $(+18,-18)_{2S}$. Cyclic loading was applied on the plates. The ratio between the minimal stress and the maximal stress was equal to 0.1. Different values of the maximal stress were studied.
The model was applied to the open hole plates. Failure of the laminates structures was defined by the criterion (15). A first ply failure strategy was applied. Experimental data and numerical simulations are compared in Figures 7 and 8.
The model matched quite well the experimental tests, which means that both damage and stress concentration have to be taken into account to predict the failure of laminated structures in the case of tensile fatigue loading.

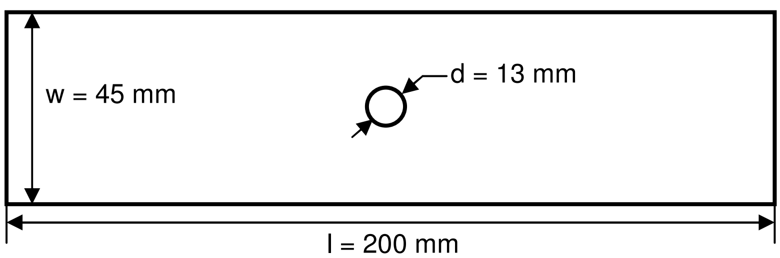

Fig.6: Geometry of the open hole plate

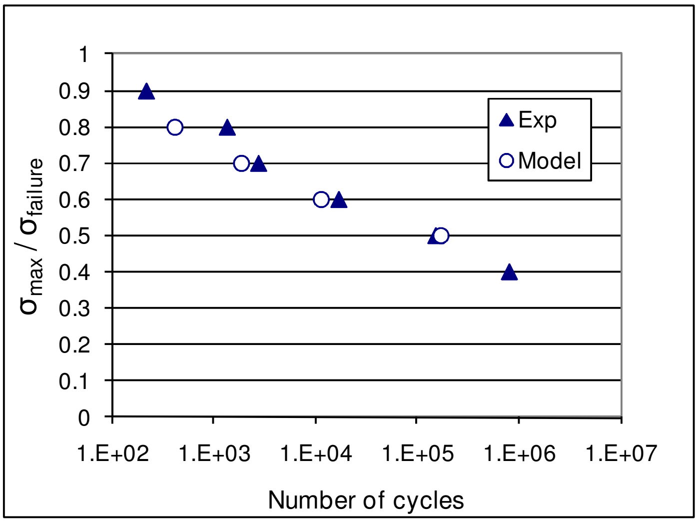

Fig.7: Comparison of experimental and numerical S-N curves for (QI) open hole plates

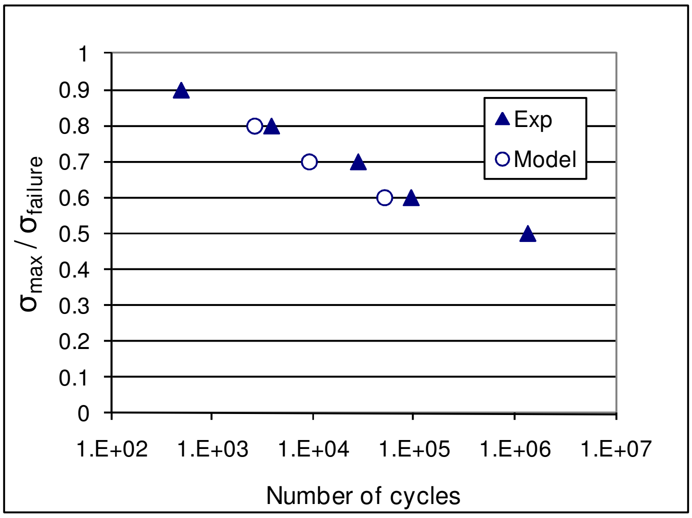

Fig.8: Comparison of experimental and numerical S-N curves for $(+18,-18)_{2S}$ open hole plates

5. Conclusion

A CDM model which describes the behaviour of laminates under static and fatigue loadings was developed and implemented in Abaqus/Standard. The simple criterion based on a Fracture Characteristic Volume previously developed to predict the failure of laminated structure under static loading was extended to the case of fatigue loading. The effects of the matrix damage and the stress concentration were investigated and taken into account in a new criterion. Experimental tests on open hole plate were performed. The results obtained with the model matched the experimental data fairly satisfactorily.
The relation between the damage, the stress concentration and the tensile fibre failure needs to be studied with more accuracy. The size of the FCV could be defined as a function of the damage. The identification of FCV size was done on a test with high number of cycles and gave a different result from the case of static loading. That means there is a strong relation between the damage and the stress concentration. In order to use the same model for static and fatigue loadings and obtained

more accurate failure predictions on fatigue, a law which governs the size of the FCV according to the level of damage would be an interesting tool.

Références

[1] Ladevèze P., A damage computational method for composite structures, *Computers & Structures* 1992;44:79-87.
[2] Thollon Y. and Hochard C. A general damage model for woven fabric composite laminates up to first failure, *Mechanics of Materials* 2009;41:820-827.
[3] Hochard C. And Thollon Y. A generalized damage model for woven ply laminates under static and fatigue loading conditions, *Int. J. Fatigue*, 2009, 32(1):158-165.
[4] Hochard, C., Lahellec, N., Bordreuil, C.. A ply scale non-local fibre rupture criterion for CFRP woven ply laminated structures. *Composite Structures* 2007;80:321-326.
[5] Miot S., Hochard C. and Lahellec N. A non-local criterion for modelling unbalanced woven ply laminates with stress concentrations, *Comp. Struct*., 2010, 92(7):1574-1580
[6] Ladevèze, P., LeDantec, E. Damage modeling of the elementary ply for laminated composites. *Composites Science and Technology* 1992;43:257-267.
[7] Withney J., Nuismer R., Strain gradient in composite laminate structure, *Journal of composite materials* 1976;35:733-735.
[8] Hochard Ch., Miot S., Lahellec N. et al., Behaviour up to rupture of woven ply laminate structures under static loading conditions, Composites: Part A 2009;40(8):1017-1023., *Composite Structures*, Vol. 92, Issue 7, 2010, Pages 1574-1580